M-Theory Phenomenology

and See-Saw Mechanisms[*]

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Abstract

A version of M-theory phenomenology is proposed in which the symmetry is based on the group $SO(10) \times SO(10) \times SO(10) \times U(1) \times U(1)$. Each $SO(10)$ group acts on a single generation. The $U(1) \times U(1)$ is regarded as the hidden sector symmetry group. The supersymmetry is broken in the hidden sector by the Fayet-Iliopoulos D-term for each group. The D-term is needed also to circumvent the powerful non-renormalization theorem since the $SO(10) \times SO(10) \times SO(10)$ is broken down to the usual $SO(10)$ by the pair condensation of certain messenger sector multiplets. The exchange of $U(1)$ gauge bosons gives an attractive force for the pair to be created and condensed. The off-diagonal mass matrix elements among the generations in these messenger sector multiplets are the source of the flavor dynamics including the CP violation. The pair condensation of another multiplet in the messenger sector leads to the doublet-triplet splitting. The $SO(10)$ decuplet Higgs couples only to one of the generations. The other couplings should, therefore, be calculated as higher order corrections. We present our preliminary results on the calculation of the mass matrices and the mixing angles for leptons and quarks in this model.

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1 Introduction

There are two significant facts which imply the necessity of going to ultra-high energy.

(1) Renormalization group

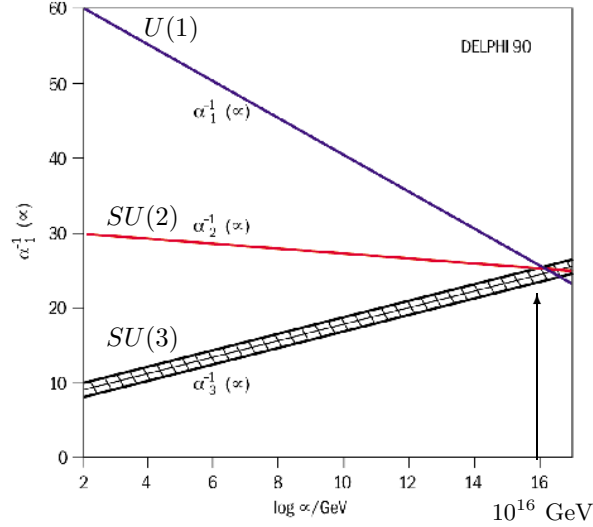As is shown in Fig. 1, the unification scale is as large as 10^{16} GeV [1].

Figure 1: Running gauge coupling constants.

(2) See-saw mechanism

This is the theme of this workshop and it was suggested 30 years ago by Gell-Mann, Ramond and Slansky [2] and by Yanagida [3] to explain the smallness of the neutrino mass. If we take the standard model scale to be $m = 100$ GeV and the neutrino mass to be 1 meV, the Majorana mass is as large as 10^{16} GeV:

$$\frac{m^2}{M} \sim \frac{(100 \text{ GeV})^2}{10^{16} \text{ GeV}} = 10^{-12} \text{ GeV} = 1 \text{ meV} . \tag{1}$$

These do not necessarily imply the Planck mass, but they are close enough. We might say that the string theory is vaguely implied by these facts since we know that the string theory seems to be the only candidate at this stage to unify the gravity with other interactions.

I note that the older generation of physicists (for example, P. Ramond) worked both in string theory and in phenomenology. But the new generation (apart from a few exceptions) works either in string or in phenomenology. I would say that this is a very unhealthy condition! Here I would like to start from the M-theory and try to go all the way down to calculate the quark-lepton mass matrices, mixing angles and so on. The present work is partly motivated by a paper of E. Witten [4].

The talk is organized in the following way. I will first very briefly touch on the subject of M-theory. Although the current conventional wisdom is that there is no consistent way to formulate it as the quantum membrane theory, I would like to challenge this. The mathematical techniques I will rely on are some classification methods of topology of 3-manifolds, such as the Heegaard diagram [5] on one hand, and the remarkable conjecture by Thurston [6] which tells us that the geometry of a 3-manifold is

unique, given its topology, on the other. These powerful techniques at least encourage us to investigate the possible existence of the renormalizable and unitary quantum membrane theory.

Next, I will switch to the phenomenology by fixing the vacuum state of the 11-dimensional supergravity theory, taking into account the following facts:

1. There are three generations of quarks and leptons.

2. The $SO(10)$ symmetry is the correct gauge symmetry.

We combine these facts with the theoretical consequences of M-theory:

1. Compactified space is the 7-manifold of G_2 holonomy [7] which presumably has the structure of $K_3 \times \Omega$ where Ω is a certain 3-manifold and K_3 is a 4-manifold with $SU(3)$ holonomy.

2. M-theory gives a natural explanation of the "deconstruction" [8] which means that the symmetry group of K_3 can be reached only from isolated points in Ω.

An explicit model with the symmetry of $\{SO(10)\}^3 \times \{U(1)\}^2$ [9] is constructed in section 4. Each $SO(10)$ corresponds to each of the three generations of quarks and leptons. $\{U(1)\}^2$ is thought to be the gauge symmetry of the hidden sector. We explicitly specify the particle content of the hidden sector and the "messenger sector" which is defined to be a set of particles which transform non-trivially, both in the physical and the hidden sector groups. One of the most important ingredients here is the fact that the $SO(10)$ decuplet Higgs couples only to one of the generations. The rest of the couplings should be calculated as higher order corrections. This enables us to calculate the quark-lepton mass matrices and the mixing matrices, starting from the first principles.

Section 5 is devoted to the discussions on the broken symmetries and the hierarchy issues. The $\{U(1)\}^2$ for the hidden sector is introduced for the following purposes:

1. Break the supersymmetry in the hidden sector using the Fayet-Iliopoulos mechanism [10].

2. Break the $\{SO(10)\}^3$ down to $SO(10)$ by dynamically breaking the symmetry with the condensation of the Cooper pairs of messenger sector multiplets.

We circumvent the powerful non-renormalization theorem [11] by using the Fayet-Iliopoulos D-term. One of the hierarchies is provided by the masses and couplings in the original Lagrangian, presumably calculated using the techniques of the membrane instanton [12]. Other kinds of hierarchical mechanisms are also important in our model. The see-saw mechanism is one example. Another mechanism is given by the hierarchical nature of the BCS-NJL order parameter [13]. In fact, the doublet-triplet splitting is the consequence of this mechanism in our model combined with the possible membrane instanton calculations.

Section 6 describes the calculation of the mass matrices and the mixing angles of quarks and leptons which I am carrying out with S. Pakvasa [14]. Although this work is still in progress, we can already see some of the interesting features of the model. The Wolfenstein parameterization for the quark mixings and the bi-maximal nature of the neutrino mixings can naturally be seen in our model. Part of CP violation can come from the non-zero phase of the BCS-NJL order parameters. The detailed calculation is underway.

2 M-theory

M-theory is a theory which is supposed to reduce to the 11-dimensional supergravity theory in the low energy regime. The matrix formulation exists [15] which seems to be a renormalizable theory as long as the compactified space is appropriate [16]. We know this theory is equivalent to the membrane theory in the classical case [17]. Can the membrane theory be quantized as a renormalizable theory? The conventional wisdom seems to be negative. The membrane theory, then, is regarded as an effective theory, just as the other n-dimensional theories with $n \geq 3$. I am not quite satisfied with this situation, since the 3-manifolds which come into play in the membrane theory have a lot in common with the 2-manifolds. The most notable examples are:

(1) The topology of the closed 3-manifolds can be classified [18] (but not uniquely) by a method which seems to be an extension of the genus expansion in the 2-manifold case, i.e. the Heegaard expansion.

(2) There seems to be a unique geometry up to homeomorphism for a 3-manifold of a given topology, i.e. Thurston conjecture [6].

These two facts (one is still a conjecture) are at least very encouraging for tackling the problem of renormalizable membrane theory.

Now, the closed membrane vacuum amplitude is given by

$$\langle 0|S|0 \rangle = \sum_{\substack{\text{all closed} \\ \text{3-manifolds}}} \int \mathcal{D}g_{ij}\mathcal{D}X_i \exp\left(-\int \mathcal{L}d^3\sigma\right) \ , \tag{2}$$

with g_{ij} the metric of a 3-manifold and X_i the coordinate for the manifold in the 11-dimensional space-time. The sum over all closed manifolds means over both topology and geometry. The topological aspect has been investigated in my earlier paper [19]. We introduce the idea of topology and geometry of 3-manifolds in what follows.

2.1 Topology

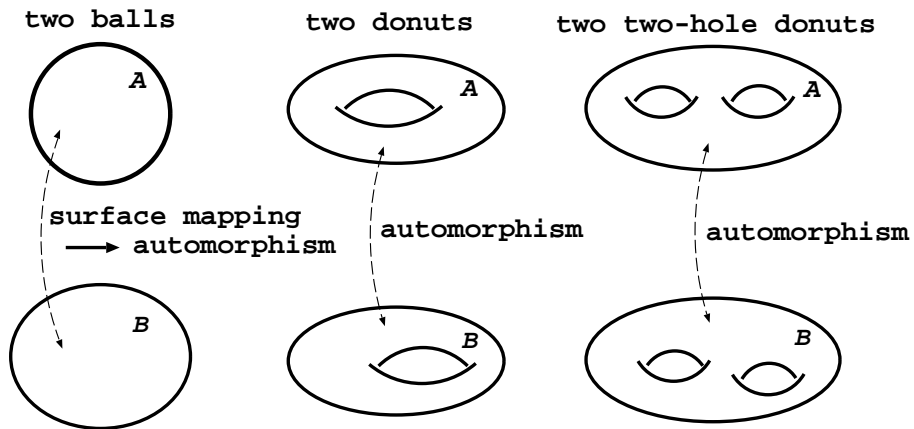Heegaard diagram: any 3-manifold can be expressed in one or more than one Heegaard diagrams [5].

Figure 2: Heegaard diagrams

Fig. 2 shows some examples of the Heegaard diagrams. Here the classification of the surface automorphism gives rise to the classification of topology of the 3-manifolds. The following are the two simple cases:

1. Two-balls

$$S^3 \; : \; x^2 + y^2 + z^2 = 1$$
$$(A) \; : \; x^2 + y^2 = 1 - z^2 \leq 1, \; z \geq 0$$
$$(B) \; : \; x^2 + y^2 = 1 - z^2 \leq 1, \; z \leq 0$$

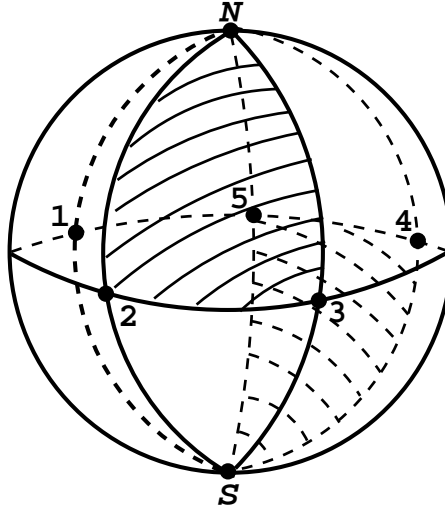2. Two donuts $\rightleftharpoons$ lens space $L(p, q)$ [20]

Figure 3: Lens space $L(5, 2)$. There are 5-sections each in upper and lower hemisphere. We identify the surface of the upper n (modulo 5)-th section with the surface of the lower $(n + 2)$ (modulo 5)-th section.

2.2 Geometry

Thurston conjecture:

> "Any geometry can be brought into the following 8 geometries by a diffeo(homeo)morphism:
> H^3, S^3, E^3, $S^2 \times S^1$, $H^2 \times S^1$, Sol, , Nil, and $SL(2, R)$ "

There is no need to integrate over g_{ij} in eq. (2). This situation is similar to the string case where g_{ij}-integration is reduced to a finite number of moduli integrations for a given topology. This is a very encouraging situation towards the proof of the renormalizability.

Unitarity, however, seems to be an outstanding problem in this approach. We are not sure yet how this problem can be tackled. One way could be to go to a limit where one can apply the duality argument to utilize the string result.

3 Low-energy effective action

11-dimensional supergravity [21] as it stands has a huge "vacuum degeneracy", which may be related to the supersymmetry of the theory. This implies that we should look for a non-generic M-theory which admits only restricted $g_{\mu\nu}$ or $A_{\mu\nu\rho\kappa}$ leading to the unique vacuum (mirror symmetry(?) [22]). The uniqueness of the vacuum does not happen in the case of string theory where the conformal invariance leads to the Einstein Lagrangian and nothing more.

Question: Does the phenomenologically correct vacuum exist among the possible vacua? If this is the case, we may work backwards to find a principle to get this vacuum as a unique solution. Many authors have worked [23] on the scheme in which

$$11 \;=\; \underbrace{4}_{\text{Minkowski}} \;+\; \underbrace{7}_{\substack{\text{compactified manifold} \\ \text{of } G_2 \text{ holonomy}}}\,,$$

$$7 \;=\; \underbrace{4}_{K_3 \text{ manifold}} \;+\; 3\,.$$

The first is the consequence of supersymmetry and the second is the consequence of mirror symmetry. The four-dimensional space (K_3 manifold) has a singularity group (monodromy group) to produce a given set of gauge particles. This gauge symmetry is localized in the 3 space, as was first shown by Katz and Vafa [24], and extensively discussed later by Acharya and Witten [25]. The supersymmetry may also be localized in the 3 space, although there is no rigorous proof on this. The proof could proceed similarly to the case of gauge symmetry.

Assumption about the vacuum

For phenomenological purposes let us assume the followings:

1. The 4 space (K_3 manifold) gives $SO(10)$ group symmetry.

2. The 3 space is a lens space $L(4,2)$ with certain discrete symmetries.

We note that this kind of model will have a dual formulation [26] in the string models which utilize the D-branes.

♠ Lens space $L(4,2)$

We use $L(4,2)$ as is shown in Fig. 4. The idea is to obtain 3+1 fixed points where 3 corresponds to three generations and 1 to the hidden sector. For this purpose we impose the following discrete symmetries to the localizing equation of the symmetry.

$$\begin{cases} (1) & y \to -y \quad \text{and} \\ & (y=0 \text{ surface and } B \text{ are invariant}) \\ (2) & z \to -z \\ (3) & x \to -x \end{cases}$$

A, B, C, O in Fig. 4 are the only fixed points under the combined symmetry of the above (1), (2) and (3). We assume that the theory is invariant under these transformations and the symmetry is localized in these points in the following way: The points A, B, and C each have a localized $SO(10)$ symmetry
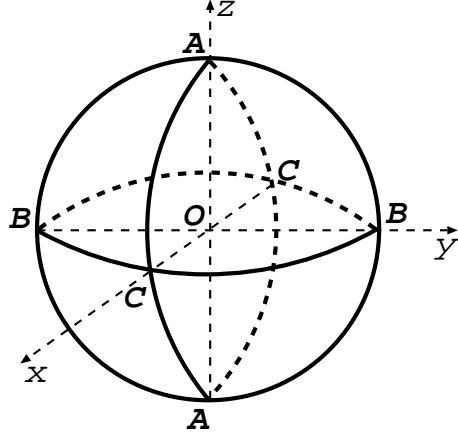
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Figure 4: Lens space $L(4, 2)$.

and the point O has the $U(1) \times U(1)$ symmetry. The discrete symmetry (1), (2) or (3) is the artifact of low-energy theory, just like all the other global symmetries, such as P, CP, baryon number, lepton number and so on. This implies that we are working on the theory not at 10^{19} GeV, but rather at 10^{16} GeV or lower.

4   Explicit model

We now construct an explicit model based on the following multiplets:

(1) Chiral multiplets:

$$Q_{16}^{(i)} \ (i = 1, 2, 3) \quad 1 \leftrightarrow A,\, 2 \leftrightarrow B,\, 3 \leftrightarrow C$$
$$H_{10}^{(1)} \text{ only at } A$$
$$H_{45}^{(i)} \ (i = 1, 2, 3)$$

Let us call these the *physical sector chiral multiplets*. $Q_{16}^{(i)}$ is the quark-lepton of the i-th generation and $H_{10}^{(1)}$ is the usual Higgs. Note that it couples only to the first generation (the 3rd generation in the usual terminology). We could have introduced $H_{10}^{(2)}$ and $H_{10}^{(3)}$ and forbid the coupling by the discrete symmetry, but we have no compelling reason to do so.

(2) Gauge fields:

$$SO(10) \ : \ V_{45}^{(i)} \ (i = 1, 2, 3)$$
$$U(1)_y \times U(1)_z \ : \ V_y \text{ and } V_z$$

(3) "Hidden" sector (point O) $U(1)_y \times U(1)_z$ chiral multiplets:

$$H(y_i), H(-y_i) \quad (i = 1, 2, 3)$$
$$H(z), H(-z),$$

where y_i and z stand for $U(1)_y$ and $U(1)_z$ charges, respectively. The interpretation of these multiplets is that there could be others, but we identified those which couple to the physical sector through messengers.

(4) Messenger sector multiplets:

$$Q^{(i)}_{16}(y_i), \quad Q^{(i)}_{\overline{16}}(-y_i) \quad (i = 1, 2, 3)$$

$$H^{(1)}_{10}(z), \quad H^{(1)}_{10}(-z)$$

These transform non-trivially, both in $SO(10)_i$ and in either $U(1)_z$ or $U(1)_y$.

Discrete symmetry

The chiral multiplets are supposed to transform in the following way under the product transformation of $x \to -x$, $y \to -y$ and $z \to -z$:

$$Q^{(i)}_{16} \to e^{i\alpha^{(i)}_{16}} Q^{(i)}_{16} \quad \text{with} \quad \alpha^{(i)}_{16} = -\pi/4 \,,$$

$$H^{(1)}_{10} \to e^{i\alpha_{10}} H^{(1)}_{10} \quad \text{with} \quad \alpha_{10} = \pi/2 \,,$$

$$\text{and} \quad H^{(i)}_{45} \to H^{(i)}_{45} \,.$$

The hidden sector multiplets transform trivially:

$$H(y_i) \to H(y_i) \,,$$

$$H(-y_i) \to H(-y_i) \,,$$

$$H(z) \to H(z) \,,$$

$$\text{and} \quad H(-z) \to H(-z) \,.$$

The messenger sector multiplets transform in the following way:

$$Q^{(i)}_{\overline{16}}(-y_i) \to e^{i\alpha^{(i)}_{\overline{16}}(-y_i)} Q^{(i)}_{\overline{16}}(-y_i) \quad \text{with} \quad \alpha^{(i)}_{\overline{16}}(-y_i) = \pi/4 \,,$$

$$Q^{(i)}_{\overline{16}}(y_i) \to e^{i\alpha^{(i)}_{\overline{16}}(y_i)} Q^{(i)}_{\overline{16}}(y_i) \quad \text{with} \quad \alpha^{(i)}_{\overline{16}}(y_i) = -\pi/4 \,,$$

$$H_{10}(z) \to e^{i\alpha_{10}(z)} H_{10}(z) \quad \text{with} \quad \alpha_{10}(z) = -\pi/2 \,,$$

$$\text{and} \quad H_{10}(-z) \to e^{i\alpha_{10}(-z)} H_{10}(-z) \quad \text{with} \quad \alpha_{10}(-z) = +\pi/2 \,.$$

The gauge multiplets are all invariant under these transformations.

Superpotential

We now write down all the possible terms in the superpotential.

$$
\begin{aligned}
W = \; & g Q^{(1)}_{16} Q^{(1)}_{16} H^{(1)}_{10} \\
& + g_i Q^{(i)}_{16} H(y_i) Q^{(i)}_{\overline{16}}(-y_i) + h H^{(1)}_{10} H(-z) H^{(1)}_{10}(z) \\
& + f_i Q^{(i)}_{16}(y_i) H^{(i)}_{45} Q^{(i)}_{\overline{16}}(-y_i) + f_{10} H^{(1)}_{10}(z) H^{(i)}_{45} H^{(1)}_{10}(-z) \\
& + m_i Q^{(i)}_{16}(y_i) Q^{(i)}_{\overline{16}}(-y_i) + m_{10} H^{(1)}_{10}(z) H^{(1)}_{10}(-z) \\
& + M_i H(y_i) H(-y_i) + M_0 H(z) H(-z) \,.
\end{aligned}
\tag{3}
$$

Here the gauge group indices are not written explicitly. The first term is the only coupling among the physical sector multiplets. The second line describes the coupling of the physical multiplets $Q^{(i)}_{16}$ or $H^{(1)}_{10}$ to the hidden sector multiplets $H(y_i)$ or $H(-z)$ through the corresponding messenger sector multiplets $Q^{(i)}_{\overline{16}}(-y_i)$ or $H^{(1)}_{10}(z)$, respectively. The third line is the coupling of $H^{(i)}_{45}$ to the messenger multiplets. The fourth line is the mass term for the messenger multiplets and the fifth line is the mass term for the hidden multiplets. We note the following:

1. The second and the third generations of quarks and leptons do not couple to the Higgs multiplet $H_{10}^{(1)}$ directly.

2. No mass term is possible for $H_{10}^{(1)}$.

3. $H_{45}^{(i)}$ couples only to $Q_{16}^{(i)}(y_i)Q_{16}^{(i)}(-y_i)$ and $H_{10}^{(1)}(z)H_{10}^{(1)}(-z)$ for $i = 1$.

4. Discrete symmetry will not be used for the explanation of doublet-triplet splitting. This will be explained later.

5. All the coupling constants can be calculated in principle in terms of membrane instantons [12]. The most important property we expect from this computation, which we assume here, is the smallness of m_{10} and M, which should be of the order of TeV, compared to the other masses which are of the scale of 10^{16} GeV or higher. This is needed to explain the doublet-triplet splitting in our scheme.

5 Broken symmetry and hierarchy

We have to consider two kinds of symmetry breaking:

(1) Supersymmetry breaking, and

(2) $SO(10) \times SO(10) \times SO(10) \rightarrow SU(3) \times SU(2) \times U(1)$.

5.1 Supersymmetry breaking

We assume that D-terms for $U(1)_z$ and $U(1)_y$ break the supersymmetry. The other motivation to include the D-terms in our model is to circumvent the non-renormalization theorem which would have resulted in [27]

$$\langle Q_{16}^{(i)}(y_i)Q_{\overline{16}}^{(j)}(-y_j)\rangle \propto \delta_{ij} \ .$$

This would have made it impossible to break $\{SO(10)\}^3$ down to $SO(10)$ through the mechanism of the pair condensation of $Q_{16}^{(i)}(y_i)Q_{\overline{16}}^{(j)}(-y_j)$. The non-renormalization theorem also would have lead to

$$\langle H_{10}(z)H_{10}(-z)\rangle = 0 \ .$$

This would have made Higgs mass vanish even in the non-perturbative calculation in our approach. The existence of D-terms is a simple way to circumvent this non-renormalization theorem.

Explicitly, we get for the D-term of the form $\kappa_y D_y + \kappa_z D_z$ with

$$\frac{|M_i|^2}{g_y y_i} < \kappa_y < \frac{|m_i|^2}{g_y y_i}, \quad \frac{|M_0|^2}{g_z z} < \kappa_z < \frac{|m_{10}|^2}{g_z z} \ ,$$

we get

$$\langle H(z)\rangle_F = M_0\langle H(-z)\rangle_A \neq 0, \quad \text{and} \quad \langle H(y_i)\rangle_F = M_i\langle H(-y_i)\rangle_A \neq 0.$$

This leads to the following consequences:

1. The $g_i Q_{16}^{(i)} \langle H(y_i) \rangle_F Q_{\overline{16}}^{(i)}(-y_i)$ term in the superpotential gives a squark mass proportional to $g_i \frac{\langle H(y_i) \rangle_F^2}{m_i}$. This means that all squark and slepton masses are of the see-saw type.

2. The $h H_{10}^{(1)} \langle H(-z) \rangle_A H_{10}^{(1)}(z)$ term gives a Higgsino mass matrix of the form:

$$
\begin{pmatrix}
0 & \langle H(-z) \rangle_A \\
\langle H(-z) \rangle_A & m_{10}
\end{pmatrix} .
$$

This is not necessarily a see-saw type because we assume m_{10} is small (TeV). The gaugino mass of $V_{45}^{(i)}$ is given by the diagram in Fig. 5.

Figure 5: Gaugino masses.

Other physical sector masses can be calculated in the following way:

1) Higgs masses can be calculated using the diagram shown in Fig. 6 with appropriate supersymmetry breaking and the symmetry breaking taken into account to evade the nonrenormalization theorem. As we will see later, the internal masses are of the order TeV for the doublet component of $H_{10}^{(1)}$ and of the order 10^{16} GeV for the triplet, giving the doublet-triplet splitting.
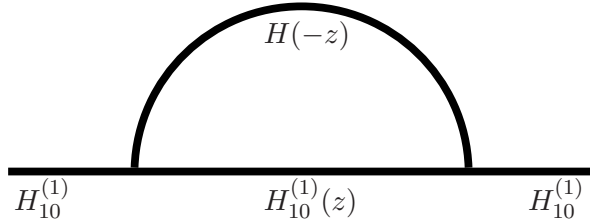

Figure 6: Higgs masses.

2) Fig. 7 gives only the $\bar{\nu}_R^c \nu_R$ mass term because we do not violate the $SU(3) \times SU(2) \times U(1)$ till the vacuum value of Higgs doublet is taken into account. The masses of quarks and leptons are the subject of section 6.

5.2 Gauge symmetry breaking

The $\{SO(10)\}^3$ breakdown to $SO(10)$ will be achieved by having

$$
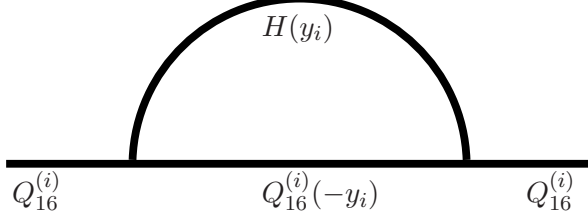\langle Q_{16}^{(i)}(y_i) Q_{\overline{16}}^{(j)}(-y_j) \rangle = m_{ij} .
$$

Figure 7: ν_R mass diagram.

The D-term can break the supersymmetry and it will also break the powerful non-renormalization theorem. In a way, this is the origin of "flavor physics". For this purpose the non-perturbative effect must be considered in the sense of Nambu-Jona-Lasinio (NJL) or Bardeen-Cooper-Shrieffer (BCS). Self-consistent mass generations à la NJL or BCS can be expressed by the following equation:

$$m_{ij} = m_i \delta_{ij} + \Sigma(m_{ij}), \quad \text{which comes from},$$

$$\Gamma(\varphi) = S(\varphi) + \frac{1}{2} \mathrm{tr} \log \left[\frac{\delta^2 S}{\delta\varphi_i \delta\varphi_j} \Big/ \frac{\delta^2 S}{\delta\varphi_i \delta\varphi_j} \Big|_{\varphi=0} \right],$$

where Γ is the one-particle irreducible potential and φ stands for the generic field.
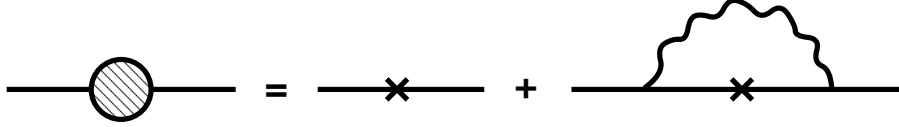

Figure 8: Self-consistent mass.

It is interesting to note that when we break $SO(10)$ down to $SU(3) \times SU(2) \times U(1)$, we need an effective cubic term for the H_{45} potential. We have

$$\mathrm{tr}\left(H_{45}^{(1)} H_{45}^{(2)} H_{45}^{(3)} \right) \neq 0 ,$$

due to the diagram in Fig. 9. This is very different from the single $SO(10)$ case where we need to add other multiplets of higher representations [28].
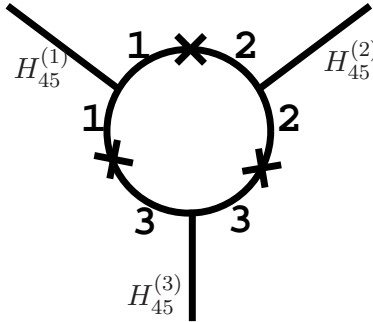

Figure 9: Cubic term.

In our case, the self-consistent mass equation (gap equation) takes the form

$$\Gamma_{\text{mass}}(\varphi) = S(\varphi) - \frac{1}{2} \mathrm{Tr} \int d\Omega d\Omega' \left[K^{-1}(\Omega - \Omega')_{\varphi\varphi} a_{\varphi V}(\Omega') K^{-1}(\Omega' - \Omega)_{VV} a_{V\varphi}(\Omega) \right.$$

$$\left. + K^{-1}(\Omega - \Omega')_{\varphi V} a_{V\varphi}(\Omega') K^{-1}(\Omega' - \Omega)_{\varphi V} a_{V\varphi}(\Omega) \right] ,$$

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where[1]

$$d\Omega \equiv dx d^2\theta d^2\bar\theta, \quad \frac{\delta^2 S}{\delta\varphi_i \delta\varphi_j} \equiv \underbrace{K_{ij}}_{L_0} + \underbrace{a_{ij}}_{L_{\text{int}}}.$$

For example,

$$K_{V\varphi} = \begin{pmatrix} -\Box P_T + \frac{1}{2}m_V^2 - \xi(P_1 + P_2)\Box & -2gY A^\dagger(y) & 0 \\ 0 & -\frac{m}{4\Box}DD & 1 + gY D\theta^2\bar\theta^2 \\ -2gY A(-y) & 1 - gY D\theta^2\bar\theta^2 & -\frac{m}{4\Box}\bar D\bar D \end{pmatrix}, \qquad \text{etc.}$$

Rather than tackling this complicated equation here, I would like to solve some prototype equations where supersymmetry is completely neglected just to illustrate the basic idea involved. The equation is shown in Fig. 10.

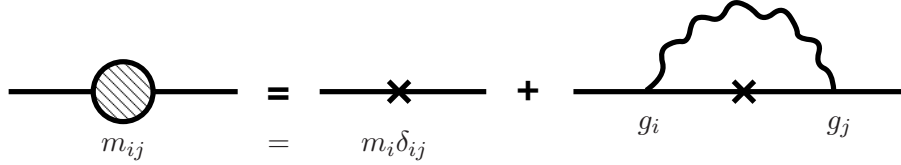$$m_{ij} \qquad = \qquad m_i\delta_{ij}$$

Figure 10: Simplified self-consistent equation.

(1) $H_{10}^{(1)}$ case

This is a single channel case and is very simple:

$$m = m_0 + g^2 m \log \frac{\Lambda}{m},$$

$$m_0 = m\left(1 - g^2 \log\frac{\Lambda}{m}\right), \quad \text{or} \quad m = \frac{m_0}{1 - g^2 \log\dfrac{\Lambda}{m}}.$$

The solution is illustrated in Fig. 11. For $m_0 = 0$, we get

$$m = 0 \quad \text{or}, \quad m = \Lambda e^{-1/g^2}.$$

The point is that the scale Λ of the broken symmetry solution $m = \Lambda e^{-1/g^2}$ has nothing to do with the scale of the bare mass m_0. The symmetry is combined with the diagram in Fig. 6. When the symmetry is broken down to $SU(3) \times SU(2) \times U(1)$, we may arrange the coupling in such a way that the doublet corresponds to the unbroken solution and the triplet corresponds to the broken solution $(m \simeq \Lambda e^{-1/g^2})$.[2] This, combined with the diagram in Fig. 6, is our mechanism of the doublet-triplet splitting.

(2) $Q_{16}^{(i)}(y_i)$ case

The most important question in this case is whether we can have a solution with the off-diagonal elements. We have

$$m_{ij} = m_0\delta_{ij} + g_i\left(m\log\frac{\Lambda}{m}\right)_{ij} g_j \qquad (i = 1, , 2, 3),$$

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[1]The notations here come from J. Wess and J. Bagger, *"Supersymmetry and Supergravity"*, Princeton University Press, 1992.

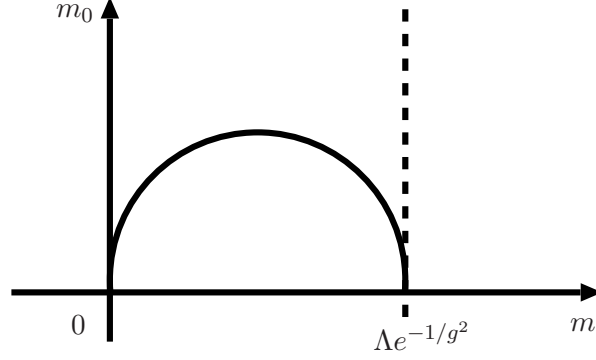[2]For $\Lambda \sim 10^{19}$ GeV and $m \sim 10^{16}$ GeV, we have $g^2 \sim 0.1$.

Figure 11: One channel solution.

where no summation over i, j is implied. We consider the case where,

$$m_{ij} = m_0 \delta_{ij} + \overline{m}_{ij}, \quad |\overline{m}_{ij}| \leq |m_0|.$$

We use the notation

$$x_{ij} = \frac{\overline{m}_{ij}}{m_0}, \quad \lambda \equiv \log \frac{\Lambda}{m_0}, \quad \text{and} \quad |g_1| > |g_2| > |g_3|, \quad \lambda \gg 1.$$

Then, the equation becomes

$$x_{ij} = \lambda g_i g_j \delta_{ij} + (\lambda - 1) g_i g_j x_{ij} - \frac{1}{2} g_i g_j \left(x^2 \right)_{ij}.$$

We get, as the approximate solution,

$$x_{11} \simeq 1/g_1.$$

Assuming

$$g_2 \geq \frac{1}{(\lambda - 1)g_1 + 1/2}, \quad g_3 \leq \frac{1}{(\lambda - 1)g_1 + 1/2},$$

we get

$$x_{22} \simeq \lambda g_2^2, \quad x_{33} \simeq \lambda g_3^2,$$

and

$$x_{12}^2 = \frac{4}{g_1 g_2 g_3^2} \left\{ 1 - (\lambda - 1)g_1 g_3 + \frac{1}{2} g_1 g_3 \left(x_{11} + x_{33} \right) \right\},$$

$$x_{13}^2 = \frac{4}{g_1 g_3 g_2^2} \left\{ 1 - (\lambda - 1)g_1 g_2 + \frac{1}{2} g_1 g_2 \left(x_{11} + x_{22} \right) \right\},$$

$$x_{23}^2 = x_{12}^2 x_{13}^2 \cdot \frac{1}{2} g_2^2 g_3^2,$$

where x_{12} is real, and x_{13} and x_{23} are imaginary, implying a contribution to the CP violation. We see that the solution with the off-diagonal element exists and at least a part of CP violation may come from the phase of the BCS-NJL order parameter.
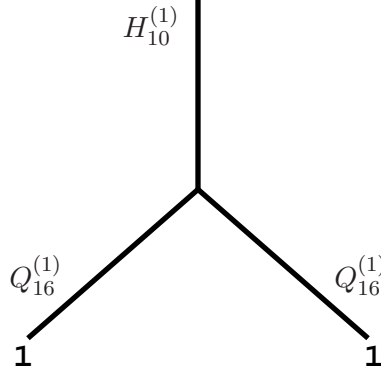
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Figure 12: Higgs coupling.

6 Quark-lepton mass matrix and mixing matrix calculation

(This part of the work is in collaboration with S. Pakvasa)

The $gQ_{16}^{(1)}Q_{16}^{(1)}H_{10}^{(1)}$ as in Fig. 12 is the only Higgs coupling in our model.

Other couplings can be calculated in the following way:

(1) $Q_{16}^{(1)}Q_{16}^{(2)}H_{10}^{(1)}$ or $Q_{16}^{(1)}Q_{16}^{(3)}H_{10}^{(1)}$ can be calculated as in Fig. 13.
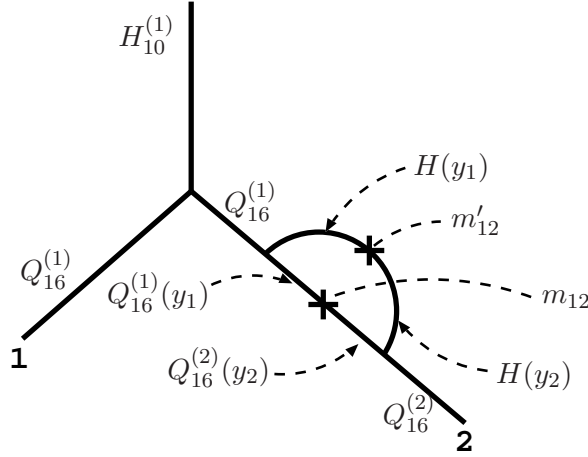

Figure 13: The first order couplings for $Q_{16}^{(1)}Q_{16}^{(2)}H_{10}^{(1)}$. $Q_{16}^{(2)}$ is replaced by $Q_{16}^{(3)}$ to calculate the third generation coupling.

(2) $Q_{16}^{(2)}Q_{16}^{(3)}H_{10}^{(1)}$ can be calculated as in Fig. 14 (a) and (b). It is easy to show that the diagram of Fig. 15 does not exist.

Mass matrices for up-type quarks U, down-type quarks D and charged leptons L, therefore, have the following form:

$$\begin{pmatrix} 1 & c & d \\ a & \alpha & \gamma \\ b & \beta & \delta \end{pmatrix},$$
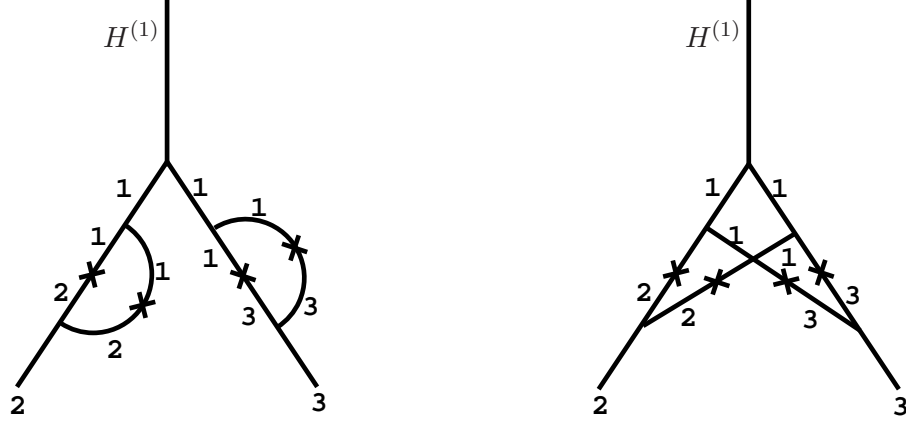
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Figure 14: (a) The second order couplings (external correction type). (b) The second order couplings (vertex correction type).
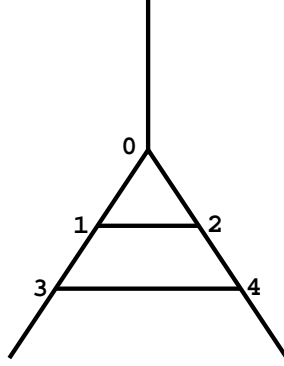

Figure 15: Diagram of this kind does not exist.

with a, b, c and d being the first order and α, β, γ and δ being the second order.

The ν_R mass matrix has the form:

$$
m_D = \begin{pmatrix} \Delta_{11} & & \\ & \Delta_{22} & \\ & & \Delta_{33} \end{pmatrix} ,
$$

taking into account the smallness of the off-diagonal elements suggested by the prototype computation in the previous section.

After some computation we get the following KM matrix[3]:

$$
\begin{pmatrix}
1 & -(\chi_-^\dagger, V_{+d}) - \bar{\kappa}_{+-}^d + \bar{\kappa}_{+-}^u & -\bar{\kappa}_{+0}^d + \bar{\kappa}_{+0}^u + \bar{\kappa}_{-0}^d(\chi_-^\dagger, V_{+d}) \\
(\chi_-^\dagger, V_{+d}) + \kappa_{+-}^d - \kappa_{+-}^u & 1 - \frac{1}{2}|\kappa_{-0}^u - \kappa_{-0}^d|^2 & -\bar{\kappa}_{-0}^d + \bar{\kappa}_{-0}^u \\
\kappa_{+0}^d - \kappa_{+0}^u - \kappa_{-0}^u(\chi_-^\dagger, V_{+d}) & \kappa_{-0}^d - \kappa_{-0}^u & 1 - \frac{1}{2}|\kappa_{-0}^u - \kappa_{-0}^d|^2
\end{pmatrix} ,
$$

where

---

[3]The matrix here is in the grand unified scale. The renormalization effect can be calculated [29].

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$$V_+ = \begin{pmatrix} 1 \\ a \\ b \end{pmatrix}, \quad V_- = \frac{1}{\sqrt{|a|^2 + |b|^2}} \begin{pmatrix} |a|^2 + |b|^2 \\ -a \\ -b \end{pmatrix},$$

$$\text{and} \quad V_0 = \frac{1}{\sqrt{|a|^2 + |b|^2}} \begin{pmatrix} 0 \\ -\bar{b} \\ \bar{a} \end{pmatrix},$$

with the suffix u or d signifying the up- or down-type quark quantities. We also have

$$\chi_\pm \equiv V_{\pm u} - V_{\pm d},$$

$$\kappa_{ik} \equiv \frac{1}{\lambda_i - \lambda_k} \langle V_k | H_1 | V_i \rangle, \quad H_1 = \begin{pmatrix} 0 & c & d \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix},$$

$$\lambda_+ = 1 + |a|^2 + |b|^2 + |c|^2 + |d|^2, \quad \lambda_- = \frac{(|a|^2 + |b|^2)(|c|^2 + |d|^2)}{1 + |a|^2 + |b|^2 + |c|^2 + |d|^2},$$

$$\text{and} \quad \lambda_0 = \frac{1}{|c|^2 + |d|^2} \left\{ |a\alpha - c\gamma|^2 + |d\beta - c\delta|^2 \right\} + \text{"second order"}.$$

The neutrino mixing matrix is also calculated to be:

$$\text{mixing matrix} = \begin{pmatrix} \cos\theta \frac{\lambda_+^*}{|\lambda_+|} & \cos\theta(a^*y - b^*x) & \cos\theta(a^*x^* + b^*y^*) \\ \sin\theta \frac{\lambda_+^*}{|\lambda_+|} & \cos\theta \cot\theta(-a^*y + b^*x) & \cos\theta \cot\theta(-a^*x^* - b^*y^*) \\ \frac{1}{N_+} \cot\theta(-by - ax) & \cot\theta(-by - ax) & \cot\theta(-bx^* - ay^*) \end{pmatrix}$$

with the neutrino mass matrix being:

$$M = m_D \begin{pmatrix} \Delta_{11}^{-1} & & \\ & \Delta_{22}^{-1} & \\ & & \Delta_{33}^{-1} \end{pmatrix} m_D^t$$

Here $\cos\theta = 1/\sqrt{1 + |a|^2 + |b|^2}$. The a and b in this formula correspond to mass matrix elements of the leptons. We also have for the neutrino parameters

$$\lambda_+ \simeq 1 + c^{*2} + d^{*2}, \quad x \simeq -\frac{1}{N_0} \left\{ b + c\beta + d\delta \right\},$$

$$\lambda_- \simeq -\frac{N_0^2}{1 + c^2 + d^2}, \quad y \simeq \frac{1}{N_0} \left\{ a + c\alpha + d\gamma \right\},$$

$$\text{and} \quad N_+^2 = \frac{|\lambda_+|^2}{N_0^2} + |x|^2 + |y|^2,$$

with a, b, etc. corresponding to the left-handed neutrino mass matrix elements. Actually, in this formula for the neutrino parameters we replaced the original parameters in the following way:

$$c^2 \Delta_{11} \Delta_{22}^{-1} \longrightarrow c^2, \quad d^2 \Delta_{11} \Delta_{33}^{-1} \longrightarrow d^2,$$

$$\alpha^2 \Delta_{22}^{-1} \longrightarrow \alpha^2, \quad \beta^2 \Delta_{22}^{-1} \longrightarrow \beta^2,$$

$$\gamma^2 \Delta_3^{-1} \longrightarrow \gamma^2, \quad \text{and} \quad \delta^2 \Delta_{33}^{-1} \longrightarrow \delta^2.$$

This means that the contribution of the Majorana mass is to renormalize the original a, b, etc.. The actual computation of the a, b, etc. through the Feynman diagrams of Fig. 13 and Fig. 14 is underway. We hope to present the results soon. Interestingly enough, our expressions for the mixing angles, even without the diagrams computation, already show some nice features: both quark and lepton mixing matrices are consistent with experiments.

7   Conclusions

(1) The effort to formulate the M-theory as the renormalizable membrane theory is continued.

(2) The vacuum problem of the M-theory should be considered both from the theoretical and the phenomenological aspects. It is important to make sure that it contains a physically acceptable vacuum, thus requiring the necessity of phenomenology.

(3) A candidate vacuum is considered, which is a kind of lens space with the discrete symmetries.

(4) $\{SO(10)\}^3 \times (U(1) \times U(1))$ model is proposed as the candidate for the phenomenology.

(5) Various hierarchy mechanisms are considered.

    1. membrane instanton
    2. see-saw
    3. BCS-NJL order parameter

(6) CKM and neutrino mixing are calculable as is explicitly demonstrated.

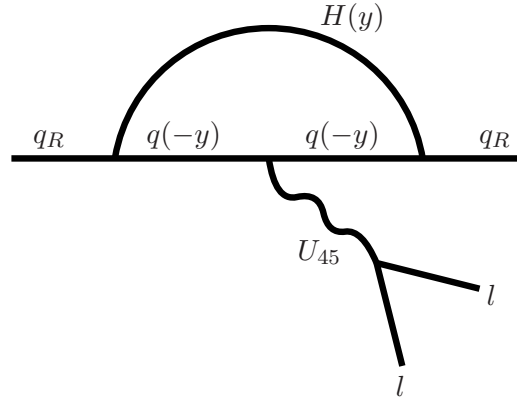(7) New physics may show up in, for example, B-physics as is shown in Fig. 16.

Figure 16: A possible new physics.

Acknowledgment

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